Pythagorean Theorem

Grade: 8
Unit: Patterns and Relations
SCO: Students should be able to demonstrate an understanding of the Pythagorean relationship, using models.

Materials (for each pair of students):

- One square piece of grid paper in each of the following sizes (appendix A):
  - 1 cm x 1 cm
  - 2 cm x 2 cm
  - 3 cm x 3 cm
  - 4 cm x 4 cm
  - 5 cm x 5 cm
  - 6 cm x 6 cm
  - 7 cm x 7 cm
  - 8 cm x 8 cm
  - 9 cm x 9 cm
  - 10 cm x 10 cm
  - 11 cm x 11 cm
  - 12 cm x 12 cm
  - 13 cm x 13 cm
  - 14 cm x 14 cm
  - 15 cm x 15 cm
- Handout of right angle (appendix B)
- Scotch tape
- Handout of spreadsheet (appendix C)
- Handout of puzzle (appendix D)
- Scissors

Advance preparation:

- Cut out one set of paper grids for each pair (appendix A), marking the area of each on the back
- Place each set of grids in a plastic zip-lock bag.
- Print one copy of the right angle (appendix B), one copy of the spreadsheet (appendix C), and one copy of the puzzle (appendix D) for each pair of students.
- Have a quick look at appendix E to see the solution to the puzzle you will be handing out.

Instructions:

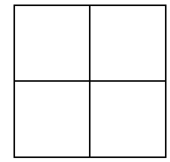
- Try to introduce the lesson in a way that provides context for the students. You may use any suitable real-life example, but I will use that of the 3-4-5 rule in carpentry. Explain to the students that carpenters need to make sure that some things that they are building have right angles. They have a precise way of checking angles called the 3-4-5 rule. They measure 3 units from the corner in one direction, and 4 units in the other direction from the corner. They then draw a line connecting the points to form a triangle. If the line measures 5 units then they know that they have made a right angle. Today we will be discovering why this rule works.
- Start the lesson by reminding students about right angles and right angle triangles (Draw a picture of a right angle triangle on the board). Explain that the two shorter sides of the right triangle are denoted as “a” and “b”, and that the longest side (the hypotenuse) is denoted as “c”

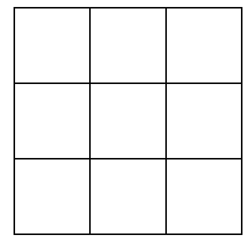
- Explain that today we will be looking for a relationship that describes the area of squares which have the sides of the right triangle as their bases. Draw a square on each side of the triangle that you drew previously on the board.
- Review with the students how to find the area of a rectangle (base x height) and that the area of a square would be $base^2$. This is a good chance for them to demonstrate previous knowledge so rather than giving them the formula, ask if anyone recalls it.
- Distribute the materials (except for the puzzle) to the class and ask them to start working in pairs by taping the handout with the right angle to their table. Ask them how do they know that this is a right angle? Could this easily be turned into a right triangle? How?
- Explain that they will be using the square grids to construct right triangles of different sizes and that when they think they have found 3 squares that match up to form a right triangle, they can record the areas of the squares in the appropriate place on the handout (appendix c). To make sure that their triangles are right triangles, they should construct them by lining the first 2 squares up on the right angle handout, and finding a third square to form the hypotenuse. You may show them an example using the 3-4-5 rule from the intro, explaining that when squared, this makes 9-16-25 (tie back in to carpentry example).
- Once each pair of students has found several combinations or Pythagorean triplets, have them read them aloud while you write them on the board or else have them come up and fill in a table on the board.
- Once this is up where everyone can clearly see the results, ask if anyone sees a pattern? If there is no response, ask if they see a pattern specifically between a,b and c. Someone should notice that when we add the values from column a and b, we get the values from column c. This is the Pythagorean model: the area of the square on side a plus the area of the square on side b equals the area of the square on side c.
- Did this pattern hold true for each triplet found by the students? If not, then explain that those cases were anomalies (the squares may have shifted so that the triangle was no longer a right triangle). To prove to them that this pattern is true, have each pair of students cut out the pattern on appendix d, cut out the square marked 5 (square a), cut out the square with the dotted lines, and cut on the dotted lines to get the shapes 1 through 4 (square b). Using these shapes, they can assemble them to fill in the area of square c (the largest).
- If they are struggling to complete the puzzle, help them by suggesting that they place piece # 5 in the middle, and fill the others around it. After they are done, discuss the pattern with them again. Ask them what they have done with squares a and b. They should agree that squares a and b have both fit into square c, or in other words, that square a plus square b is the same size as square c.
- Ask the students "Do you think that this relationship would hold true if we used half-circles instead of squares?" "What about any other geometrical shape such as a triangle?" Either do a quick example with them of these cases on the board or else give them as problems to solve with their partner.

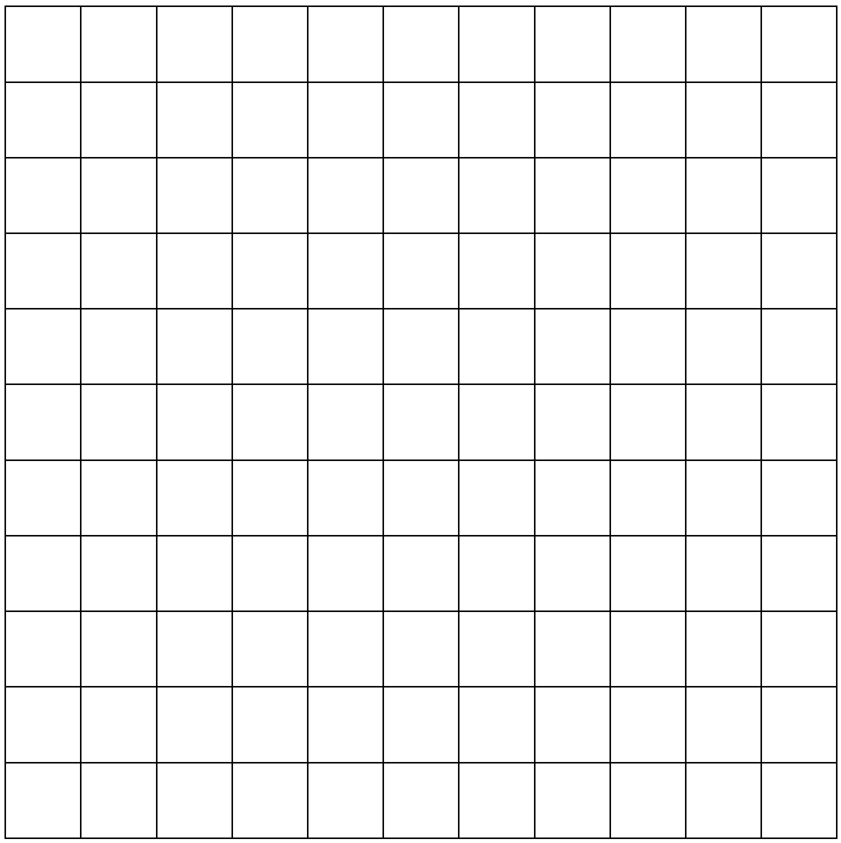
Enrichment

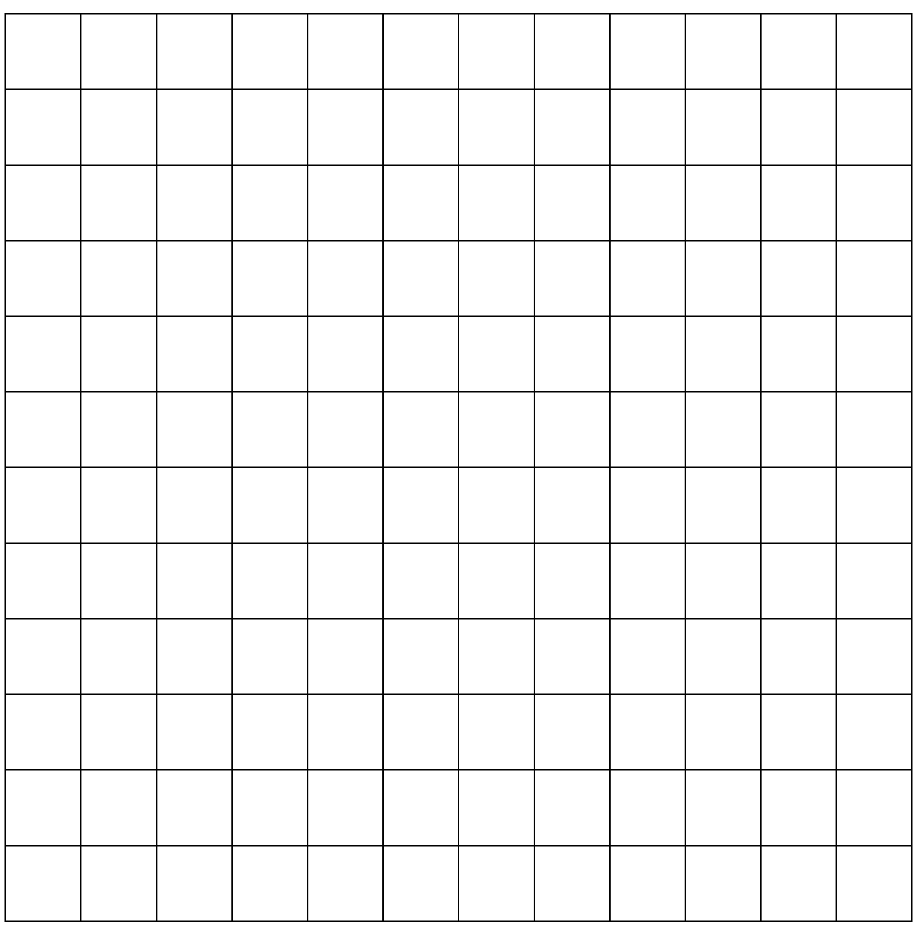
- The actual Pythagorean theorem ($a^2 + b^2 = c^2$) is not introduced until grade 10, but if you have students who seem to have a very good grasp on the concept of the Pythagorean relationship, you may offer this website as enrichment: http://arcytech.org/java/pythagoras/index.html
- If you do not wish to send the students to the website on their own, you can print out templates from the website and use them as an in-class activity.

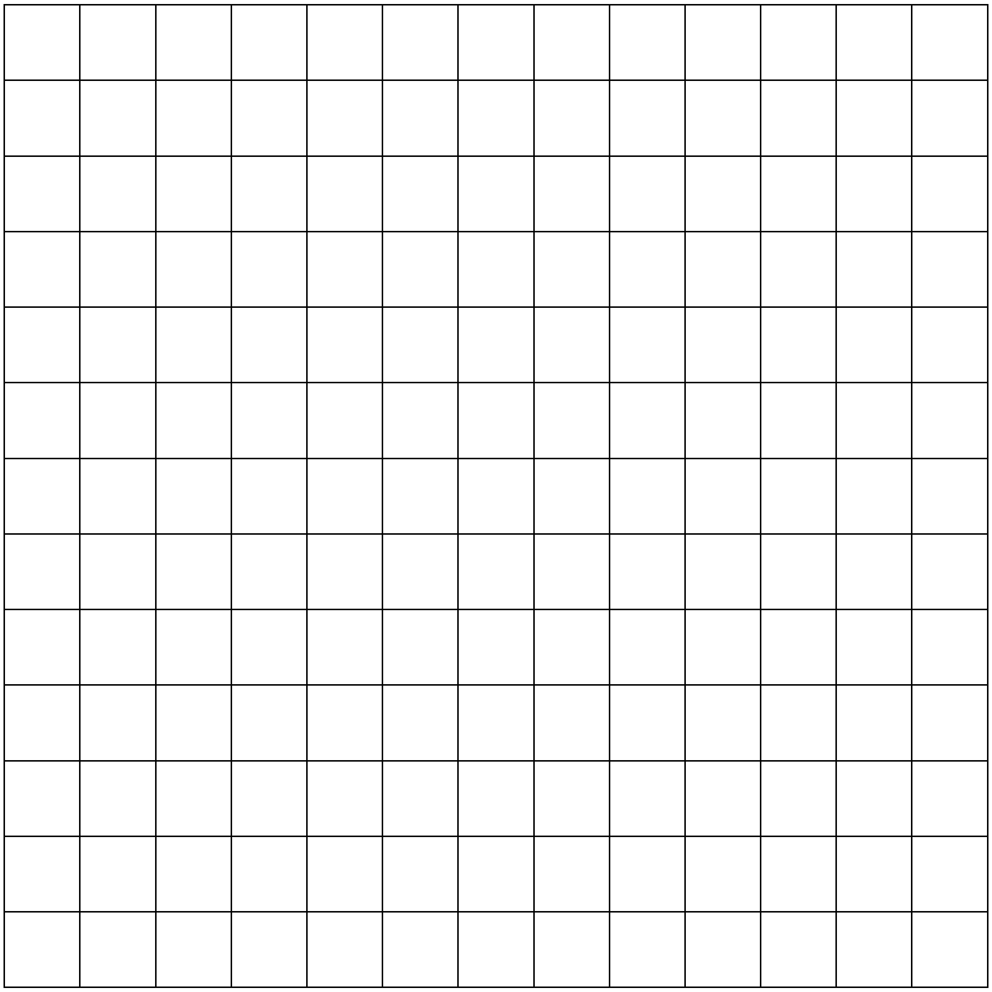
Appendix A (5 pages)

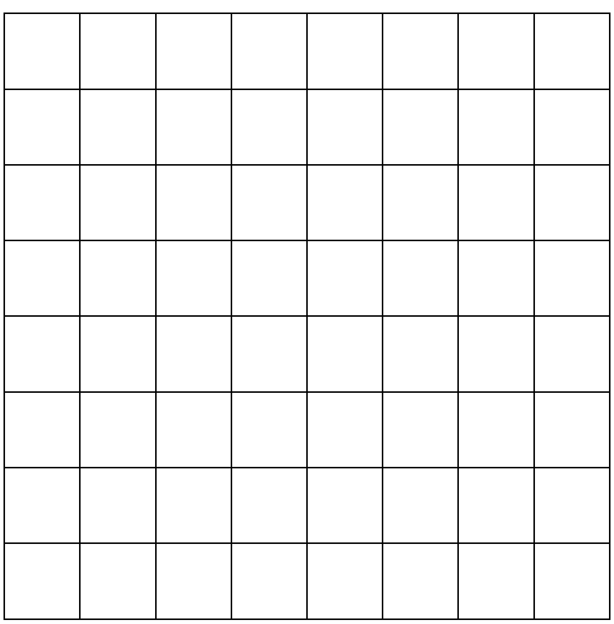


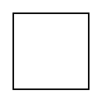


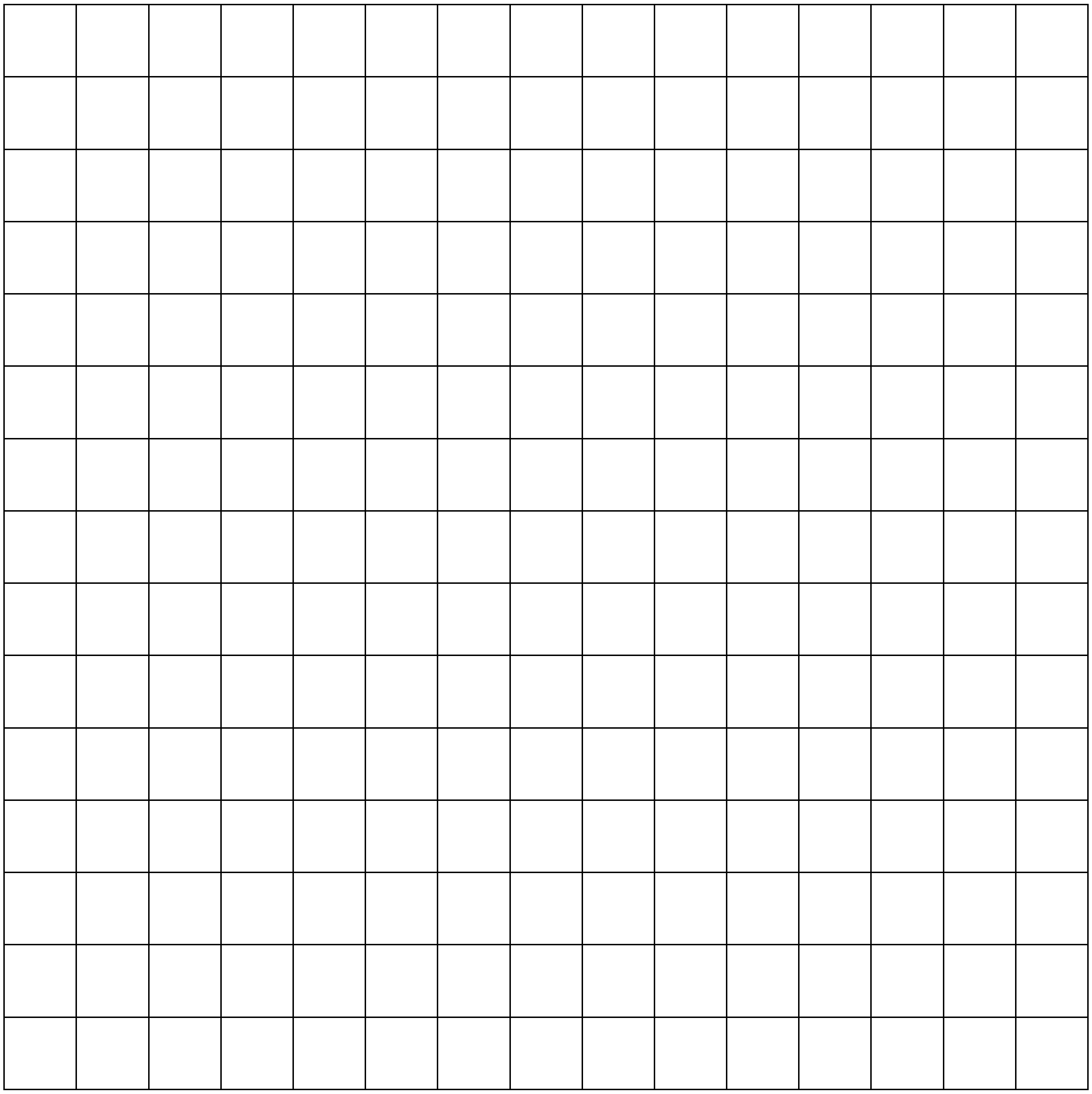


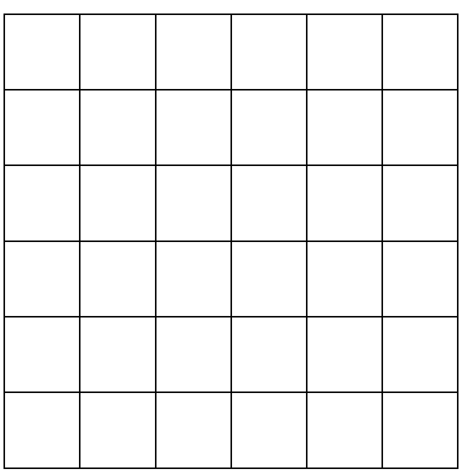




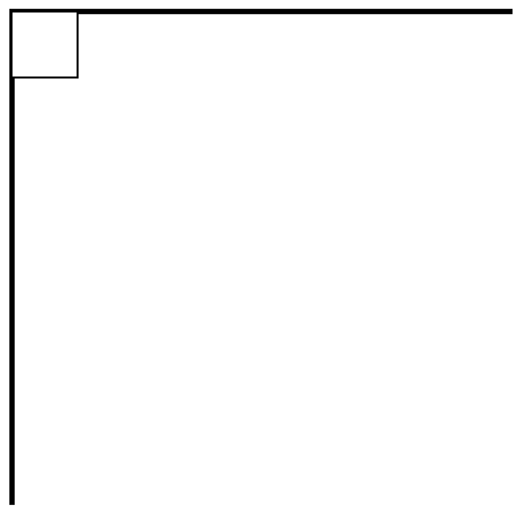








Appendix B



Appendix C

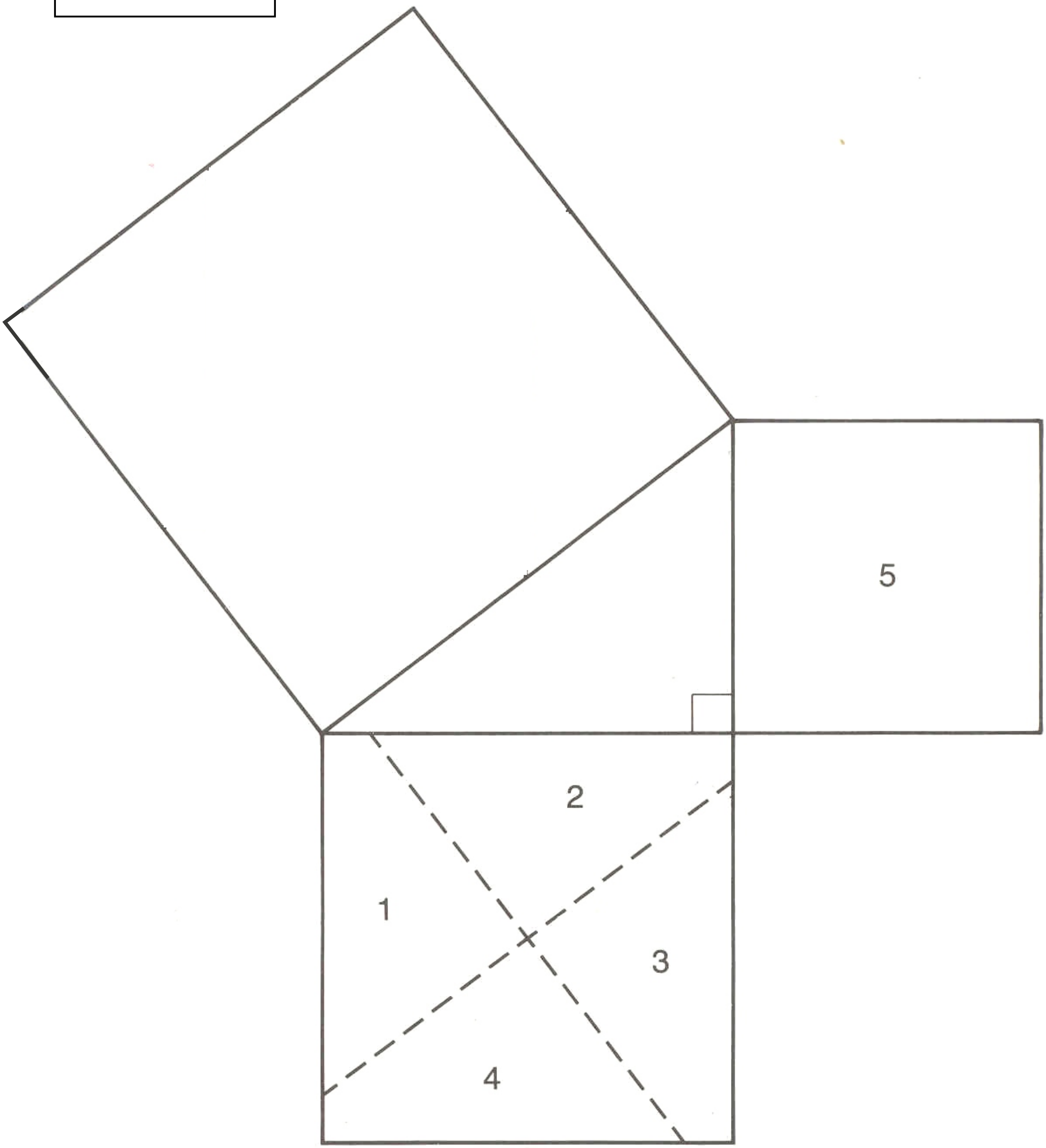
The Pythagorean Relationship

Names:

Area of square on side "a"	Area of square on side "b"	Area of square on side "c"

Based on the data collected above, can you put into words the Pythagorean relationship?

Appendix D



Appendix E